

# Mathematics Specialist Units 1 and 2 Formula Sheet

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## Trigonometry

$$\sec \theta = \frac{1}{\cos \theta}, \cos \theta \neq 0$$

$$\operatorname{cosec} \theta = \frac{1}{\sin \theta}, \sin \theta \neq 0$$

$$\cot \theta = \frac{1}{\tan \theta}, \tan \theta \neq 0$$

$$\cos^2 \theta + \sin^2 \theta = 1$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$\cot^2 \theta + 1 = \operatorname{cosec}^2 \theta$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\begin{aligned}\cos 2A &= \cos^2 A - \sin^2 A \\ &= 2 \cos^2 A - 1 \\ &= 1 - 2 \sin^2 A\end{aligned}$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$$\cos A \cos B = \frac{1}{2} [\cos(A - B) + \cos(A + B)]$$

$$\sin A \sin B = \frac{1}{2} [\cos(A - B) - \cos(A + B)]$$

$$\sin A \cos B = \frac{1}{2} [\sin(A + B) + \sin(A - B)]$$

$$\cos A \sin B = \frac{1}{2} [\sin(A + B) - \sin(A - B)]$$

In any triangle  $ABC$

Sine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Cosine rule

$$a^2 = b^2 + c^2 - 2bc \cos A$$

## Vectors

Magnitude

$$|\mathbf{a}| = |(a_1, a_2)| = \sqrt{a_1^2 + a_2^2}$$

Unit vector

$$\hat{\mathbf{a}} = \frac{1}{|\mathbf{a}|} \cdot \mathbf{a}$$

Scalar product

$$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| \cdot |\mathbf{b}| \cos \theta = a_1 b_1 + a_2 b_2$$

Vector projection of  $\mathbf{a}$  on  $\mathbf{b}$

$$\mathbf{p} = p \cdot \hat{\mathbf{b}} \text{ where } p = \mathbf{a} \cdot \hat{\mathbf{b}}$$

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## Number

Natural

$$\mathbb{N} = \{ 1, 2, 3, \dots \}$$

Integer

$$\mathbb{Z} = \{ \dots, -2, -1, 0, 1, 2, \dots \}$$

Rational

$$\mathbb{Q} = \{ x : x = \frac{a}{b}, \text{ where } a, b \in \mathbb{Z}, b \neq 0 \}$$

Real

The set of real numbers  $\mathbb{R}$  consists of the set of all rational and irrational numbers.

Complex

$$\mathbb{C} = \{ z : z = a + bi, \text{ where } a, b \in \mathbb{R}, i^2 = -1 \}$$

## Combinatorics

There are  $n(n-1)(n-2) \times \dots \times 3 \times 2 \times 1 = n!$  ways to arrange  $n$  different objects in an ordered list.

Permutations

$${}^n P_r = n(n-1)(n-2)\dots(n-r+1)$$
$$= \frac{n!}{(n-r)!}$$

Combinations

$${}^n C_r = \binom{n}{r}$$
$$= \frac{n(n-1)(n-2)\dots(n-r+1)}{r!}$$
$$= \frac{n!}{r!(n-r)!}$$

Inclusion-exclusion principle

$$n(A \cup B \cup C) = n(A) + n(B) + n(C)$$
$$- n(A \cap B) - n(A \cap C) - n(B \cap C)$$
$$+ n(A \cap B \cap C)$$

## Matrices

Determinant

$$\text{If } \mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \text{ then } \det \mathbf{A} = ad - bc$$

Inverse

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Dilation

$$\begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$

Rotation

$$\begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

Reflection

$$\begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix}$$

## Function

$$\text{If } ax^2 + bx + c = 0 \text{ then } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$